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By Thomas Blench, M. ASCE

IRRIGATION DIVISION

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PAPERS

REGIME THEORY FOR SELF-FORMED SEDIMENT-BEARING CHANNELS

By Thomas Blench,[1] M. ASCE

Synopsis

A dynamical framework—that of regime theory—on which to base a study of the behavior of channels that have formed boundaries from their own transported material or material of like nature is given. The most important system of natural channels to which the theory can be applied is that of the rivers of alluvial plains. Rivers, in general, are not susceptible to quantitative analysis except for certain broad effects. The theory has grown from the unrivaled mass of data provided by the irrigation canal systems of northern India. These canals were built by British engineers in India at the end of the nineteenth century and were extended regularly until they formed the largest composite body of canals in the world.

It is shown that canals of the type considered provide formulas for the self-formation of width, depth, and slope. Rivers require two more formulas to give the general pattern of meandering. The developed laws fit into the framework of accepted hydraulics and generalize it. Although these laws apply to rivers, exactly as to canals, the disturbing influences in the former require judgment in the application of the formulas. Self-forming rivers cannot be described adequately in terms of the one flow formula of rigid-boundary hydraulics.

Regime-Type Rivers

Rivers of alluvial plains have the peculiarity that they build their own boundary conditions out of their transported material or out of material transported by previous rivers. In detail the action is complicated; but the over-all results, when studied in terms of a suitable time scale, show a definite

Note.—Written comments are invited for publication; the last discussion should be submitted by October, 1951.

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pattern of behavior consistent with a theory of regime that will be expounded in this paper.

The term "regime" was adopted many years ago, before there was any definite theory, by irrigation engineers who dealt with canals of fairly constant discharge, excavated in alluvium and carrying a sediment load. The term, which implies both an obedience to certain laws and a considerable freedom of action, is an appropriate one. To declare that a river has acquired a regime is comparable in a general sense to stating that a territory has acquired a climate. The vagaries of the weather from day to day are great and have not yielded to exact analysis. Nevertheless, a regime of weather such as a climate is recognizable; and reasonably definite laws have been discovered, such as those causing the trade winds, those limiting rainfall, and those affecting temperature.

A regime-type river is defined as one that has formed a major part of every cross section from material that has been transported or could be transported at some stage of flow.

A river is said to be "in regime" in a reach if its mean measurable behavior during a certain time interval does not differ significantly from its mean measurable behavior during comparable times before or after the given interval.

These definitions will help to simplify the discussion of river behavior. A few examples of classification will be given:

(*a*) Suppose that a record of river gages for twenty years, corresponding to a discharge of 200,000 cu ft per sec, is plotted against time. If this "specific gage" record shows a definite trend, then the river is not in regime, but the behavior is consistent with its classification as a "regime type." If the record shows no trend, the river is practically certain to be in regime during the period for all factors provided it is of regime type as previously defined.

Regime implies a time scale. A record of a few weeks would be worthless, and to require a record of centuries would probably disqualify every channel on earth from passing the test for regime. This peculiarity has caused considerable controversy in the more exact applications of the term. Actually there is no need for controversy since the difficulty arises from a lack of sense of proportion. All measurements of physics imply a scale. A simple example is the case of measuring the velocity of flow in a channel, using a current meter. Measurements occupying a few seconds are inconsistent because of turbulence. Measurements over a period of a few minutes are satisfactory and are now used universally because they yield values practically suitable for an average velocity. Measurements over a period of many minutes would be influenced by variations of the mean and would prove useless. There is no dispute as to the practical existence and utility of a "mean velocity," although rigorous logic might prove that no such quantity existed. As a further example of the importance of a scale, in conditioning viewpoint, engineering lengths would not be measured by a scale designed to indicate molecular subdivisions.

(*b*) Suppose that observations were made monthly for several years along the logitudinal sections of a main line canal presumed to be in regime and that they always indicated practically the same result. This is remarkable behavior for a regime canal, and the chances are that the section is not of regime type. It would probably be lined with stiff clay without a sand cover. Beds of

regime channels usually oscillate seasonally about equilibrium. Channels excavated in rigid material and sections that prevent deposition or scour are not of regime type. However, an entirely cohesive boundary may have become stable after erosion and have no sand deposit, in spite of sand being available in suspension, because the section is too constricted to permit deposition. This case is unusual, and, if the observer did not know that the scour had occurred, he would be justified in classing the channel as a nonregime type if he were interested in channels with sand beds. If he were interested in the history of clay-boundary channels carrying sand-free water and if he knew that scour had occurred, he would classify the channel as being of regime type. A rock channel would always be nonregime when free from deposition.

(*c*) Suppose surveys of a considerable length of river were available every year for thirty years, and suppose the mean meander length of fully developed meanders were calculated from plans every year. If the first fifteen-year mean came to 1 mile plus or minus a standard deviation of 20%, and if the second fifteen-year mean came to 1.2 miles plus or minus 15%, it would indicate that these particular calculations are consistent with regime so far as meandering is concerned. (The question whether such a test makes the best use of the statistics does not matter for the present illustration.)

(*d*) Suppose that a torrential river has a shingle bed that does not move except in high flood. In low stage the shingle is masked by very fine sand. Such a river is of regime type. It can be said to be in one regime when running low and moving the sand by saltation and in another regime when in spate (freshet flow) with the sand in suspension and the shingle rolling. To decide whether the river is in regime the best measure would be to record "specific gages" at different points. Practically, it would not matter much whether the gages were for high stage or low stage. Such a river would not be found in alluvial plains as they are generally defined.

Enough has been said to indicate the use of the term "regime" and to show that even simplified cases may introduce complications. The question now arises as to how much is known, without special regime theory, of behavior that would indicate the existence of simple laws. The following statements should meet with no objection:

(1) Osborne Reynolds observed[2,3] that, in regime-type rivers, small discharge meant relatively great depth with respect to width, and he concluded that the exaggeration in the slope of small channels was the same as the exaggeration of depth-to-width ratio. This is borne out by the experience of river model experimenters, who have all found that the bed material refuses to move without vertical exaggeration;

(2) It is a general observation that, aside from the effect of material, large rivers have relatively flat slopes and great widths compared with depth;

(3) Coarse material is associated with steep slope and greater width-to-depth ratio on an equal discharge basis;

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[2] "Uniform Flow in Alluvial Rivers and Canals," by Gerald Lacey, *Minutes of Proceedings*, Inst. of Civ. Engrs., London, England, Vol. 237, 1933–34, p. 421.

[3] "Stable Channels in Alluvium," by Gerald Lacey, *Minutes of Proceedings*, Inst. of Civ. Engrs., London, England, Vol. 229, 1929–30, p. 259.

(4) Large rivers have large meanders; small rivers have small ones;

(5) The varieties of meander pattern occurring in large rivers find their counterpart in small rivers.

Apparently, simple laws are at work in regime-type rivers; yet there are many examples of engineers who fail to foresee retrogression below dams on such rivers and aggradation upstream from the reservoirs. Perhaps the best hint of the nature of one of the laws lies in the analogy between "smooth-boundary" flow in pipes and regime-type flow. In the former case a simple fluid forms its own boundary with its own substance so that the resistance law is expressible in terms of viscosity without any reference to the material of the pipe. In the latter case a water-sediment complex arranges itself so as to form a water-sand

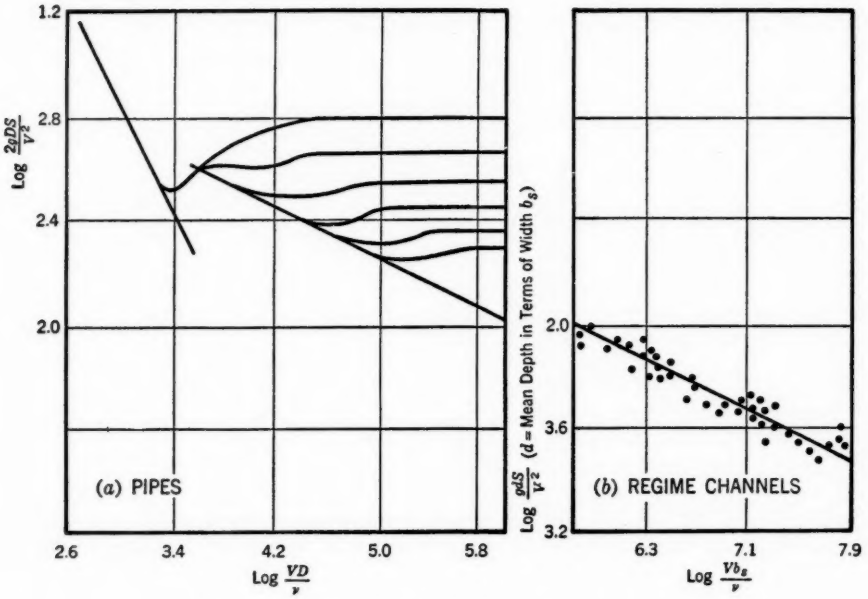

FIG. 1.—FLOW FORMULAS, GENERALIZED REGIME THEORY

bed zone of considerable complexity and sometimes to build a pair of banks of fine sand, silt, and clay. Consequently, the flow formula for regime-type rivers might be expected to be a generalized case of that for rigid "smooth" boundary pipes, and this is precisely what has been found (see Fig. 1).

GROWTH OF REGIME THEORY

The vagaries of discharge and the heterogeneous nature of banks render regime-type rivers practically useless for analysis aimed at deriving quantitative laws. This explains why regime theory originated on the great canal systems of India and Pakistan, where discharges are controlled and meandering is prevented. As these canals provide what is virtually the laboratory of regime

science, a history of the growth of the theory will be presented. Special reference is given to the canal systems of Punjab Province, India, with 70,000 cu ft per sec of authorized supply and 20,000,000 acres of irrigation. The United Provinces (India) and Sind (Pakistan) provide comparable amounts. The channels of all three provinces are mostly in alluvium and derive water from regime-type rivers without storage. The American counterpart of one of the Indian canals is the All-American Canal, Arizona-California. The Punjab, being the major irrigating province, affords most information. Major canal work started in the 1880's with two canals of 7,000 cu ft per sec and 9,000 cu ft per sec capacity. They were excavated in alluvium on the basis of the Kutter formula, using arbitrary slopes and widths. They were characterized by an excessive sedimentation—worse in the one with the lesser ground slopes—that persists to this day. These difficulties caused R. G. Kennedy[4] to study the relation between velocity and depth in some channels that gave no trouble, and he produced a formula which, unfortunately, is still quoted:

$$V = cd^N \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

in which V is the mean velocity, d is the mean depth, and c and N are variables. Mr. Kennedy produced design diagrams that give the simultaneous solutions of the Kutter equation and Eq. 1. As canals have three self-adjusting variables, he was one equation short; so engineers were still free to design erroneously but with only one degree of ignorance instead of two. In the course of time unofficial rules of thumb were produced to make up for the lack of a third equation. Mr. Kennedy was unlucky in having chosen channels of different sediment grades and side conditions; so his formula is nonrepresentative of any simple physical condition and has not been used in India since 1930.

In 1903 three more canals were completed in India with capacities of 4,500 cu ft per sec, 9,000 cu ft per sec, and 11,000 cu ft per sec, respectively. One exhibited serious sediment trouble on one branch but not on the other, because engineers did not understand regulator designs that could divide sediment suitably. Another developed very severe trouble largely because of failure to understand the effect of approach conditions on sediment distribution in the river. The third developed trouble later, in spite of initially adequate slope, because of heavy sediment entering from level crossings provided for drainage torrents. This condition has been overcome by bed ejectors. In 1914 a feeder canal with a capacity of 13,000 cu ft per sec was built to divert adjacent river water into a system with a capacity of 7,000 cu ft per sec. The feeder had adequate slope and has given little trouble, but the main line of the receiving system became scoured, and material settled in the laterals.

The result of the experiences on all these canals was a statement by E. S. Lindley,[5] Assoc. M. ASCE, crystallizing in 1919 the principles that had been forced on engineers in their losing battle against nature:

> "When an artificial channel is used to convey silty water, both bed and banks scour or fill, changing depth, gradient and width, until a state of

[4] "The Prevention of Silting in Irrigation Canals," by Robert Greig Kennedy, *Minutes of Proceedings*, Inst. of Civ. Engrs., London, England, Vol. CXIX, 1894-95, p. 281.

[5] "Regime Channels," by E. S. Lindley, *Minutes of Proceedings*, Punjab Engineering Congress, Lahore, India, Vol. 7, 1919.

balance is attained at which the channel is said to be in regime. These regime dimensions depend on discharge, quantity and nature of bermsilt, and rugosity of the silted section; rugosity is also affected by velocity, which determines the size of wavelets into which the silted bed is thrown."

Although details of the statement may be criticized, the point to notice at present is that the existence of three self-adjusting variables had been accepted, with nature choosing the magnitude of each to suit the discharge and sediment load. Mr. Lindley also stated the following results of observations made throughout the feeder canal (13,000 cu ft per sec) while he was in charge of it:

$$V = 0.95\, d^{0.57} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2a)$$

and

$$V = 0.57\, b^{0.355} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2b)$$

in which b = average width of the channel. He still accepted the Kutter formula, and apparently he did not suspect that the exponents indicated physically significant laws based on the values $\frac{1}{2}$ and $\frac{1}{3}$.

A system of canals from four diversion dams was completed in the latter half of the 1920's. Although the Kennedy design diagrams were still accepted officially, some engineers had obtained knowledge published soon thereafter and had used it unofficially so that these canals gave little sediment trouble.

The major theoretical step followed the United Provinces Government's placing Gerald Lacey on special duty to analyze data to find the laws of regime. The results are available[2,3,6,7] although not very easy to apply as written. One account that is applicable in practice is an unavailable Indian publication. The formulas have been accepted officially by the Central Board of Irrigation of the Government of India since 1934,[8] although they had considerable use before that date.

Since the introduction of the Lacey theory, the Irrigation Research Institutes of the provinces of India and Pakistan have subjected it to most searching field tests by choosing channel field sites and observing them at intervals of a few days for several years. The results vindicate the exponents in the Lacey equations but show that the constants depend on sediment grade; the single "silt factor" F is not adequate. In fact the insertion of field data into the equations derived by Mr. Lacey gives values of F that are found to be functionally related. In 1941 the writer and C. King showed a method of separating F into a bed factor and a side factor,[9,10,11] and these factors have been found to apply more consistently than the original silt factor F. This generalized Lacey theory is the one presented in this paper.

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[6] "A General Theory of Flow in Alluvium," by Gerald Lacey, *Journal*, Inst. of Civ. Engrs., London, England, Vol. 27, 1946, p. 16.

[7] "The Behaviour and Control of Rivers and Canals," by Sir Claude C. Inglis, *Publication No. 13*, Govt. of India Central Waterpower, Irrigation and Navigation Research, Yeravda Jail Press, Poona, India, 1949.

[8] Constitution of the Research Committee of the Central Board of Irrigation with a complete list of Resolutions of Research Committees up to and including the 10th Research Committee in July, 1940; Central Board of Irrigation, Simla, India, 1940.

[9] "Effect of Dyanmic Shape on Lacey Relations," by T. Blench and C. King, *Annual Report (Tech)*, Central Board of Irrigation, Simla, India, 1941.

[10] "Practical Design Formula for Stable Irrigation Channels," by C. King, *Annual Report (Tech)*, Central Board of Irrigation, Simla, India, 1943.

[11] "Theorie de L'Ecoulement Turbulent," by T. Blench, *La Houille Blanche*, N. S., No. 3, May, 1946, p. 163.

A canal system built in India in 1936 with a capacity of 5,000 cu ft per sec was designed entirely according to the Lacey theory and has given no appreciable trouble. A system twice as large (10,000 cu ft per sec) was designed similarly in 1947. The old canal systems have been remodeled considerably and with notable success according to Lacey principles since the inception of the theory.

It may be said that, nowadays, armed with regime theory, the engineer finds what the canals want to do, and he does it first. With care he can adapt the theory for rivers; but without an enormous range of canal information he never would have discovered regime theory.

Data Supporting Regime Theory

The Lacey regime theory[2,3,6] was developed from observations on both canals and rivers. The generalized theory used in this paper rests on the analysis of canal data alone, although it is noted that the behavior of rivers is consonant with the findings from canals. The findings of the Lacey theory, as supported by canals, have been accepted in general (although modified in detail) so that his data are used implicitly. Data taken from field observations of Irrigation Research Institutes of the provinces and others are used in addition. The scope and nature of the Lacey and subsequent data are given in eight items as follows:

a. The canals from which data are accepted were assumed to be in regime. Because of the various sources of information used, some of the data accidentally refer to steady but not regime sections. It should be clear from the beginning that perfect regime does not exist in nature any more than perfect equilibrium exists, but there is no doubt about the existence of a practical regime. Observations on a practical regime channel may be likened to observations of the position of a planet describing an equilibrium path with slight perturbations. Sufficient observations, when plotted, indicate the major law of equilibrium.

b. The regime canals observed possessed berms. A berm is a step in the bank, at about the elevation of full-supply level, and it exists whether the section is excavated or has deposited its own sides with engineering assistance to ensure regularity and no meandering. Common practice is to excavate the sides below berm level to a slope of 1 on $\frac{1}{2}$, this being about the inclination of self-formed sides. Sometimes sides are excavated at slopes of 1 on 1. Maintained sides acquire a slight curvature. Maintenance consists of periodic cutting back to prevent meandering (the sides usually grow in again by deposition) and cutting back of grass edges that would tend to overhang. Usually the side material is of silty clay-loam nature.

c. The beds of the regime canals observed were covered by clean sand that moved in dune or ripple formation. Fig. 2, from a model of selective deposition flume shows the high-level bed in typical formation. Apart from the dunes the bed, in straight canal reaches, was of visibly uniform elevation across any section; cross sections plotted from soundings showed a slight dishing. There is no doubt about the distinct nature of the sides and bed nor the legitimacy of referring to the depth d (depth from full-supply level to practically flat bed) instead

of a mean depth. There have been references to ideal elliptic sections; they find no support from observations on channels with cohesive sides.

d. Discharges in the records ranged from about 5 cu ft per sec to 10,000 cu ft per sec with most of the data between about 20 cu ft per sec and 500 cu ft per sec. Standard practice is to run laterals to full supply or close them. Laterals technically include those canals with discharges up to 500 cu ft per sec, but few exceed 150 cu ft per sec. Channels in excess of 150 cu ft per sec have short-supply periods, but the bed is then unlikely to be active, and there will seldom be appreciable suspended load. Some large systems operate only during the high river season; therefore, a large supply does not necessarily mean that the channel has appreciable periods of short supply. Generally, however, the data for channels with flows as great as 500 cu ft per sec may be expected to give least scatter due to variable discharge. The discharge used is usually the authorized full supply. Channels with capacities of 20 cu ft per sec or less have rather poor discharge accuracy if measured by current meter, but discharge measurement will be accurate if the channels are fitted with meter flumes.

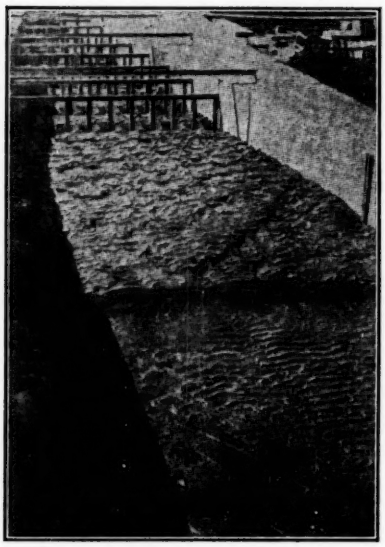

FIG. 2.—TYPICAL BED DUNES AND RIPPLES

e. The temperature variation of water was from about 50° F to 85° F, causing a considerable variation in viscosity. Turbidity caused a small but appreciable change of viscosity. Fortunately most formulas contain viscosity raised to a small fractional power.

f. The sediment grade exposed on the bed was from about 0.1 mm to 0.5 mm, the vast preponderance of sample mean values being approximately 0.25 mm. The sediment in suspension during a year varied from extremes of zero to about 1% by weight of flow and seemed to have little effect on the movement of bed material. However, suspended sediment is important in berm formation. The movement of bed material in the saltation phase is probably about $\frac{1}{100}\%$ by weight of the water flow.

g. The important Reynolds number is the one in terms of width, and it ranges from about 10^5 to 10^8. The square of the Froude number in terms of depth ranged from about 0.5 to 1.5 times $1/g$. Width to depth ratios varied between 5 to 1 and 30 to 1.

h. The data included torrents transporting material up to approximately 4-in. size; but the generalized theory does not rely on these sizes. There is no reason for excluding coarse material merely on the grounds that it does not

saltate like fine material. This peculiarity would alter the form of any relation between bed factor (defined subsequently) and size and other factors of bed material, but should not alter relations between V, d, b, slope, and bed factor. The grounds for objection are that channels with coarse material are usually those with variable discharge, and they do not present definite cohesive banks and reaches free from meandering.

If the scope and nature of the data are not appreciated, there will be timidity in extrapolating where permissible, and possibly error will be committed by extrapolating to small models where the sides mask the bed completely.

Generalized Regime Theory

In a complete exposition it would be necessary to give an account of the Lacey theory without which the present generalized theory would not exist. However, such a course would require too lengthy a dissertation. Therefore the presentation here will refer to the steps of development, with acknowledgment to Lacey theory, and will otherwise ignore that very valuable treatment. However, it should be mentioned that (presumably because of the nature of the data available from records at the time) Mr. Lacey used a single sediment factor F, thereby effectively obtaining results for a physically possible set of channels whose relative importance of side action to bed action had been "averaged out" to some arbitrary value. This permitted him to work in terms of wetted perimeter (P) and hydraulic radius (R). Mr. Lacey sought simple exponents as indicated by his plottings within the limits of statistical significance. The exponents in the Lacey formulas are suggestive of relations that have physical interpretations consistent with the findings of rigid boundary hydraulics. As the channels are generally wide compared with depth, an analysis in terms of depth (d) and mean width (b) will yield the same exponents within the limits of statistical significance. This fact justifies replacement of P and R, as used in the Lacey formula, by b and d in the generalized theory. Mr. Lacey always used P and R.

The first equation of generalized regime theory is[2]

$$\frac{V^2}{d} = B \qquad (3)$$

in which V is mean velocity of flow, d is depth from water level to flat bed, and B is a bed factor related to the nature of the bed load and constant as long as the factors relevant to bed load are constant. The formula is accepted as exact when constant discharge, constant bed load, and regime conditions have been attained. Dividing both sides of Eq. 3 by g (the gravitational constant) shows that the Froude number in terms of depth is constant when the bed load is fixed. Obviously Eq. 3 provides the means for defining a dynamically satisfying bed factor.

The formula was derived by continuing the Kennedy line of investigation (Eq. 1) with more complete and accurate data. From channels of many sizes in one system V and d were plotted logarithmically against each other. When the formula was indicated (with an exponent close to 2) for one system, then another system with an obviously different type of sediment load was analyzed,

and so on. It is noteworthy that some care must be taken in choosing channels at random within a system. Thus channels of a given discharge range must be selected from different parts of a system, for some systems have channels equipped with head regulators that take more than a fair share of bed sediment, so that the bed factor drops progressively from head to tail of the system some 150 miles away. With proper random sampling the resulting plot should give a line corresponding to an average bed factor, more or less representative of the bed load entering at the head of the main line.

The second equation of generalized regime theory is

$$\frac{V^3}{b} = s \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

in which b is chosen for convenience as the width that, multiplied by d, gives the cross-sectional area of flow. Eq. 4 defines a side factor, s, that (after some dimensional adjustment explained subsequently) measures the square of the tractive force intensity over the sides. Eq. 4, like Eq. 3, may be taken as exact for perfect regime conditions and is practically applicable to real regime conditions.

The formula was derived by theoretical argument[9,10,11] in the process of finding the reason for universal agreement on the exponent $\frac{1}{2}$ in the Lacey formula:

$$P = 2.67\, Q^{\frac{1}{2}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5a)$$

(in which Q is the discharge). The formula is equivalent to:

$$\frac{P}{R\,V} = 7.11 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5b)$$

There is serious disagreement over the constant 2.67, which obviously needed to be replaced by a variable dependent on the nature of the bed sediment. The detailed derivation of the meaning of Eq. 4 requires a little more than simple hydraulics and will be found in Appendix I. The line of reasoning is as follows:

Universal acceptance of the exponent $\frac{1}{2}$ in Eq. 5a, based on data covering a very wide range (see, for example, Fig. 3), suggests that the expression $P/V\,R$ has a physical significance. Replacing P and R by b and d, the expression can be written as:

$$\frac{V^2/d}{V^3/b} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (6)$$

in which the numerator has physical significance. Therefore there should be physical significance in the denominator. Since V^2/d is a force per unit mass, V^3/b (which is associated with the sides) is probably in some way a measure of tractive force on the sides. Appendix I shows that $\rho^2\,\nu\,V^3/b$ measures the square of the tractive force intensity on the sides (except for a multiplying constant) provided that there is a laminar film at the sides (that is, provided that the sides are technically "smooth"). Therefore, the presumption is that the

sides of most of the canals observed were "smooth" and that the Lacey formula expressed nothing more than an averaging out of the relative importance of bed to sides as measured by the ratio B/s. That is, all Lacey channels are regime channels, but not all regime channels are Lacey channels.

Logically, s would be better replaced by vs. Practically, this step is not desirable. The tractive force intensity on the sides of a regime channel may lie anywhere between that preventing the deposition of material in suspension to that which scours material already in place. A considerable range of values of s is possible without violating the condition of regime. Furthermore, viscosity varies considerably, and it is hard to assess when water is turbid. There is no practical loss in using s in design.

The third equation of generalized regime theory is

$$\frac{V^2}{g\,d\,S} = C\left(\frac{V\,b}{\nu}\right)^{\frac{1}{4}} \qquad (7)$$

in which S is channel slope measured as a fraction, C is a nondimensional constant, and $V\,b/\nu$ is the Reynolds number in terms of width.

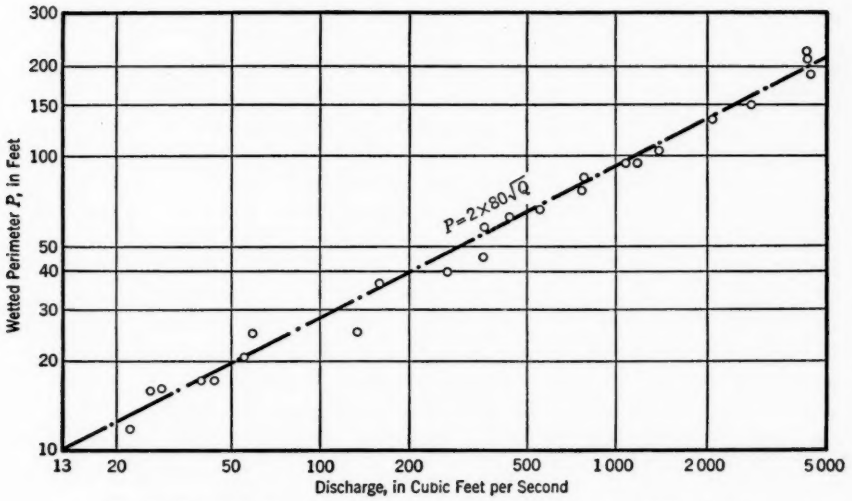

FIG. 3.—WETTED PERIMETER AS A FUNCTION OF DISCHARGE; PUNJAB DATA

Data supporting Eq. 7 are shown in Fig. 1(*b*). Plotted to the same scale, displaced to make pipe and canal data more comparable, Fig. 3(*a*) shows the data for artificially roughened pipe (D = diameter) reported by J. Nikuradse.[12] The Nikuradse lines in Fig. 1(*a*) are roughly plotted and are presented for general information only. The physical significance lies in the parallelism of the lines in the two sections of Fig. 1. Mr .King[13] made them collinear merely to empha-

[12] "Strömungsgesetze in rauhen Röhren," by J. Nikuradse, *Forschungsheft*, Verein Deutscher Ingenieure, Heft 361, 1933.

[13] "Stable Channels in Alluvium," by Gerald Lacey, *Minutes of Proceedings*, Inst. of Civ. Engrs., London, England, Vol. 229, 1929–30, opposite p. 58.

size his point that the flow formulas are similar for different boundary conditions.

It has been suggested previously in this paper that the flow formula for regime channels should be a generalized form of that for channels or pipes with smooth rigid boundary, and Eq. 7 shows such is the case. Advocates of the Prandtl-von Kármán logarithmic type of formula may dispute the applicability of the Blasius formula. The Blasius formula for pipes gives the friction coefficient

$$f = \frac{0.316}{(V\,D/\nu)^{\frac{1}{4}}} \qquad (8)$$

in which D is diameter. Recent data of large conduits for Reynolds Numbers greater than 10^6, plotted on a diagram of f against Vd/ν, contradict the Prandtl-von Kármán line markedly;[14] but they form groups consistent with the correctness of the Blasius line (the locus of Eq. 8).

Eq. 7 can be transposed into "rough boundary" form by using it in combination with Eqs. 3 and 4. It then yields:

$$V = \sqrt{C}\sqrt[4]{\frac{d}{\sqrt{\nu s/B}}}\sqrt{g\,d\,S} \qquad (9a)$$

in which $\sqrt{\nu s/B}$ is a linear dimension that may be called an "equivalent protuberance height." This shows that there is no contradiction betweee Eq. 7 and the fact that bed dunes obviously produce what is usually called "roughness." In fact, the results merely indicate that there seems to be a universal flow formula, just as there is a universal velocity distribution (to proper nondimensional coordinates). This universal formula yields the special formulas when equivalent protuberance is properly expressed. As a flow formula may be derived by integrating a velocity distribution formula, the assumption of universality of the one implies universality of the other. This idea has been put forward previously[15] and suggests that the well-known Manning formula should have the exponent $\frac{3}{4}$ instead of $\frac{2}{3}$ for R. Such a change removes the defect inherent in the Manning formula, in which n is dependent on a small fractional power of Q.

A little algebraic manipulation of Eqs. 3, 4, and 7 will yield

$$g\,V\,S = \frac{B\,(\nu\,s)^{\frac{1}{4}}}{C} \qquad (9b)$$

which shows that a regime channel adjusts itself so that the rate at which gravity does work per unit mass per unit time depends on the three factors specifying the water-sediment complex and does not depend on anything else—particularly the discharge.

Thus the three formulas of regime (Eqs. 3, 4, and 7) not only fit into the framework of conventional hydraulics but also generalize it.

Practical Regime Formulas.—Eqs. 3, 4, and 7 are unsuitable for practical use because the designer wishes to know values of b, d, and S to select for given values

[14] *Transactions*, ASCE, Vol. 113, 1948, p. 1067.

[15] "A New Theory of Turbulant Flow in Liquids of Small Viscosity," by Thomas Blench, *Journal*, Inst. of Civ. Engrs., London, England, Vol. 11, 1938–39, p. 611.

of Q, B, and s. Algebraic manipulation of the equations will give the design equations:

$$b = \sqrt{\frac{B\,Q}{s}} \quad \ldots\ldots (10)$$

$$d = \sqrt[3]{\frac{s\,Q}{B^2}} \quad \ldots\ldots (11)$$

and

$$S = \frac{B^{5/6}\, s^{1/12}\, Q^{-1/6}}{\frac{C\,g}{\nu^{\frac{1}{2}}}} \quad \ldots\ldots (12)$$

The value of $C\,g/\nu^{\frac{1}{2}}$ when ν averages 10^{-5} has been found to be 2,080.

The designer must assess the value of B that nature (restricted by the sediment exclusion and ejection actions of engineering structures) will impose on his canal system. There is no scientific formula to assist in making this assessment any more than there is a scientific formula to permit assessing a value of Manning's n or an equivalent sand-roughness size for a rigid pipe or channel. With the behavior of an existing canal known, an estimate can be made by inserting observed values of b, d, S, Q in the equations, just as a given pipe factor (n) can be found from performance and will then give an idea of what to expect from similar material. There is a rough empirical formula, adapted from an equation proposed by Mr. Lacey[16] (who was careful to emphasize its empiricism), to the effect that:

$$B = 2\sqrt{D_m} \quad \ldots\ldots (13)$$

in which D_m is the mean sample diameter of bed material (in millimeters) to be expected in the canal. Scientifically the formula is thoroughly unsatisfactory; practically it fits many cases quite well probably because the quantities in movement in saltation are very minute in most practical canal cases. If the canal is fed by a river with sand shoals, the estimation of D_m from the sand should be safe, for the expert engineer has many ways of preventing his canals from receiving the full share of river-bed sand.

The lowest value of B found in Indian canals is about 0.6; the highest value is about 1.25. The common value is approximately 1.0. These values refer to canals of the alluvial plains.

The value to assume for s is about midway between that likely to cause scour of berm material in place and that likely to permit deposition of new material on the berms from suspension. For glacial till material the designer can use 0.3; for silty clay-loam material, about 0.2; for material with little cohesion, 0.1. Sand sides of rivers below low tide level give values between about 0.05 and 0.10. The use of average values in design permits the maintenance engineer to make changes to canal widths within limits, and thereby produce permanent alterations to slopes. Although such alterations of slope are small, they can produce useful changes to command of flat land.

[16] "Uniform Flow in Alluvial Rivers and Canals," by Gerald Lacey, *Minutes of Proceedings*, Inst. of Civ. Engrs., London, England, Vol. 237, 1933–34, p. 16.

Regime Formulas as Applied to Rivers.—As the regime formulas have proved the existence of definite laws relating to the self-formation of regime-type channels, they have an application to rivers of regime type. However, an allowance for disturbing effects and variable discharges must be made in terms of engineering common sense and the formulas regarded with suspicion until supported by results. Some apparently satisfactory applications have been found to be:

(1) Determination of width between incised banks;
(2) Determination of scour between bridge piers;
(3) Determination of scour downstream of piers, along groins, and at spur heads;
(4) Aggradation upstream from reservoirs;
(5) Degradation downstream from reservoirs;
(6) Estimation of dredging;
(7) Determination of model scales.

Width Between Incised Banks.—Where a river in flood flows like a canal between incised banks and is fairly straight, the width between those banks is fairly well represented by Eq. 10 provided maximum flood discharge is used for the discharge Q. Using this Q Mr. Lacey has found[17] considerable information showing that

$$b = 2.67\, Q^{\frac{1}{2}} \qquad (14)$$

is in substantial agreement with actual conditions. Eq. 14 is also in accordance with the spans of several Indian railway bridges over large rivers that had been reduced in span during successive remodelings to suit the waterway really required. The practice of the nineteenth century was to span everything that looked like a river during flood spill. The formula is acceptable for "shingle" rivers also, because their sides are not like those of a canal. In "shingle" rivers s and B increase together, thereby making the square root of their quotient relatively unvariable. It is now considered good practice to choose the spans of bridges and barrages in Indian plains' rivers in terms of Eq. 14.

Scour Between Bridge Piers.—If the approach of a river to a bridge is fairly straight, Eq. 11 may be used to estimate the scour between the piers. It is usual to solve it in conjunction with Eq. 10 to become:

$$d = \sqrt[3]{\frac{q^2}{B}} \qquad (15)$$

in which $q = Q/b$ is estimated from the peculiarities of the river. For example, two thirds of the river discharge may be assumed to go through half the spans of the bridge. This element of judgment based on experience cannot be avoided. If the flow is oblique, the depths are doubled for design.

Scour Downstream from Piers, along Groins, and at Spur Heads.—Investigations on scour at various installations have shown that Eq. 11 fits observed

[17] "Stable Channels in Alluvium," by Gerald Lacey, *Minutes of Proceedings*, Inst. of Civ. Engrs., London, England, Vol. 229, 1929–30, p. 270 and p. 274.

behavior exceedingly well in terms of empirical coefficients. The variation with $Q^{\frac{1}{3}}$ is confirmed excellently, Q being the maximum flood discharge and d being the maximum scoured depth. Different types of structures had different coefficients.[7]

Aggradation Upstream from Reservoirs.—The backwater that occurs upstream from a reservoir causes deposition in the valley upstream because the slope is less than necessary for regime. The river tends to keep filling its valley in an attempt to regain regime. At the same time a delta advances into the reservoir, giving ever-advancing pivot points from which the river tries to set up regime slope. The ultimate fate of all such rivers is that they will fill the reservoir and run with regime slope which can be drawn back from the site of the dam. Although the ultimate fate may not occur for centuries, the rise in specific levels at the original lake head in a couple of decades can be serious.[18] There is a good record of the behavior of the rivers of the Indus system[19] as a consequence of diversion dams that have been built on it regularly since the end of the nineteenth century. Such records confirm the fact that a barrage ultimately raises the river to be parallel to its old slope up to the next diversion dam.

Degradation Downstream from Reservoirs.—Obviously the removal of bed sediment by deposition in a reservior must result in retrogression of levels downstream. It should be allowed for in the location of points of hydraulic jump on spillway floors.

Estimation of Dredging.—In 1948 comparison of the Meyer-Peter formula[20] for sediment quantity movement with the Lacey formula led to good agreement with the observed dredging quantities of several years on two rivers in the United States. The discharge used was the arithmetic mean over the time of record. That the arithmetic mean happened to be so successful is fortuitous. Obviously, the Lacey formulas do not apply for off-regime conditions, but some mean should be expected to satisfy the formulas.

Model Scales.—It is common Indian practice[7] to design the scales of river models instead of detemining such scales by the lengthy process of trial and error. Eqs. 10, 11, and 12 would be multiplied by different coefficients to allow for the variability of such factors as discharge, heterogeneity of banks and meandering. However, if arrangements are made to give about the same bed factor in model and prototype (by using the same kind of bed material and injecting suitable quantities) and if the side irregularities are reproduced faithfully in effect, the scale ratios of the coefficients will be unity, and

$$b_r : d_r : S_r = Q^{1/2}{}_r : Q^{1/3}{}_r : Q^{-1/6}{}_r \quad \ldots\ldots\ldots\ldots\ldots\ldots (16a)$$

in which the subscript r denotes the ratio of model to prototype dimension.

Eq. 16*a* explains the rule—

$$(\text{Depth scale}) = (\text{length scale})^{2/3} \quad \ldots\ldots\ldots\ldots\ldots\ldots (16b)$$

[18] "Future of Lake Mead and Elephant Butte Reservoir," by J. C. Stevens, *Proceedings*, ASCE, May, 1945.

[19] "Regime Levels on the Indus System, Part I," *Punjab Irrigation Branch Paper No. 16*, Class B, Lahore, India, 1944.

[20] *Proceedings*, Federal Interagency Sedimentation Conference, USBR, Washington, D. C., January, 1948, p. 114.

—which is sometimes stated to be empirical.[21] There is no objection to decreasing the width or using a different bed material except that Eq. 16 would then have to take cognizance of the possibility of B_r and s_r and other scale ratios related to the indefinite factors not being unity. As their exact values would not be known, the model would display difficulties before functioning effectively.

Meandering.—Excellent work has been done in studying meandering.[7,22] When disturbing causes are removed, meander pattern appears as something just as definitely characteristic of a channel as width or depth. The conclusion is that meander length and meander width can be scaled in exactly the same way as $\sqrt{Q}$ —according to observations on models and on masses of river data. The nature of the coefficients has not been determined, and unfortunately most laboratory work has been done without the aid of regime theory.

Perhaps the best indication that meander size varies as width or as $Q^{\frac{1}{2}}$ is the fact that no modelmaker has ever found that he had to distort meanders; but modelmakers who did not distort the vertical scale have all met with failure, unless they were not modeling regime-type conditions. There are two extra regime equations to define the influence of meandering in rivers in terms of meander breadth M_b and meander length M_l:

$$M_b = C_b Q^{\frac{1}{2}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (17a)$$

and

$$M_l = C_l Q^{\frac{1}{2}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (17b)$$

In Eq. 17*a*, Q is a representative discharge (usually the maximum flood discharge). The coefficient C is a function of B and of a quantity of the nature of a very disturbed side factor, s. It is also related to the form of the river hydrograph, because the selected representative discharge has to be converted into the mean that is dynamically correct for the purpose.

Conclusion

Regime theory provides a dynamic framework for the study of the behavior of rivers that form themselves from their transported materials or from similar material in place. It is precisely applicable to controlled canals of similar type.

APPENDIX I.—Tractive Force Intensity Deduced from Laminar Film Hypothesis

The first step is to find an expression for the relative thickness of the laminar film when flow is turbulent. Considering a channel in uniform turbulent flow, of depth d and of great width, the x-axis is taken in the direction of flow and the y-axis is upward at right angles to it. Then, using the conventional symbolism of hydrodynamics, the Navier-Stokes equation for the mean motion can be written as:

$$\frac{D\bar{u}}{Dt} = -\frac{1}{\rho}\frac{\partial \bar{p}}{\partial x} + X + \nu \nabla^2 \bar{u} - \frac{\partial}{\partial x}\overline{u'u'} - \frac{\partial}{\partial y}\overline{u'v'} - \frac{\partial}{\partial z}\overline{u'w'} \ldots\ldots (18)$$

[21] "Scale Models in Hydraulic Engineering," by Jack Allen, Longmans, Green Co., London, England, 1947.

[22] "A Laboratory Study of the Meandering of Alluvial Rivers," by J. F. Friedkin, U. S. Waterways Experiment Station, Vicksburg, Miss., 1945.

in which:

$\bar{u}$ = mean velocity along Ox; t = time; ρ = fluid mass per unit volume; ν = kinematic viscosity; X = body force per unit mass; and ∇^2 = Laplace operator. Eq. 18 reduces to the following in terms of the special problem: For the laminar film—

$$0 \doteq X + \nu \frac{\partial^2 \bar{u}}{\partial y^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (19a)$$

and for the fully turbulent portion of the fluid—

$$0 \doteq X - \frac{\partial}{\partial y} \overline{u'v'} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (19b)$$

In Eqs. 19 $X(= g\,S)$ is the same for all points.

It is assumed that, as suggested by the form of observed velocity distributions, there is some small distance δ from the boundary beyond which the turbulent component suddenly begins to become important compared with the viscous component of shear stress. That is, at distance δ from the boundary the turbulent component is unimportant, but at $k\,\delta$ (in which k is of the order of 1) it is important. This implies that $\left(\nu \frac{\partial^2 \bar{u}}{\partial y^2}\right)_{\delta}$ in the film is equal to $-\left(\frac{\partial}{\partial y}(\overline{u'v'})\right)_{k\delta}$ just beyond the film. The first bracketed expression can then be represented by some constant number of times V/δ^2 and the second by some constant number of times V^2/d, if it is accepted that there is a universal velocity distribution law in some system of coordinates. Because the respective results depend on the relative positions of the two points in the laminar and turbulent zones, respectively, δ and d are used.

Therefore $\nu\, V/\delta^2 \propto V^2/d$; or

$$\left(\frac{\delta}{d}\right) \propto \left(\frac{V\,d}{\nu}\right)^{-\frac{1}{2}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (20)$$

One way to see that a formula of the type of Eq. 20 is plausible is to use a variation of an argument that has been applied to establish a stability criterion that decides when laminar flow is likely to change to turbulent. This approach, however, will not reveal what happens after the breakdown occurs, which is the present problem. The extended argument is that the change from laminar to turbulent flow is likely to depend on the Reynolds number in terms of film thickness and on the relative thickness of film to depth; that is:

$$\frac{V\,\delta}{\nu}\left(\frac{\delta}{d}\right)^n \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (21)$$

If n equals 1.0, Eq. 20 is verified.

It is to be noticed that Eq. 20 is of exactly the form suggested by L. Prandtl for the related problem of a boundary layer thickness along a flat plate and does not agree with any of the several conventional forms devised to give an idea of

the magnitude of thickness of a laminar film without consideration of specific dynamical meaning.

If Eq. 20 is accepted and if it is assumed that there is a laminar film along the sides of a channel of width b, the relative laminar film thickness over the sides can be represented by $(V\,b/\nu)^{-\frac{1}{2}}$. The shear stress intensity can then be represented by:

$$\tau = \frac{\mu\,V}{\delta} \alpha \frac{\mu\,V}{b} \times \frac{b}{\delta} \alpha \frac{\mu\,V}{b} \left(\frac{V\,b}{\nu}\right)^{\frac{1}{2}} \alpha\, \rho \left(\frac{\nu\,V^3}{b}\right)^{\frac{1}{2}} \ldots\ldots\ldots\ldots (22)$$

Eq. 22 explains the physical significance given to the term V^3/b that is indicated by an analysis of the formulas derived from the data of regime canals.

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